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Efficient Filling of a School Bus via Ping Pong Balls

Solving an age-old problem.

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Abstract

How many ping pong balls does it take to fill an average school bus?

Some assumptions have to be made to make the calculation.

Due to lack of details about the interior space such as:

Curved corners of the roof, grooves or bumps in the floor, discrepancies in dimensions due to windows, lack of data regarding seats, doors, steps, gas/brake pedals, steering wheel, etc.

The interior space of the bus will be considered to be a cuboid.

Due to lack of visibility into sales metrics of different school buses to find the most common type, a model closest to the bus most commonly used when I would take the school bus as a child will be selected.

First we need to understand that there are several ways in which spheres of equal size can stack:

Simple Cubic, Body Centered Cubic, Hexagonally Close Packed (aka Face Centered Cubic)

Each type of stacking creates a different density with Simple Cubic being the least dense and Hexagonally Close Packed being the densest. As stated by the Third Law of Thermodynamics entropy in nature is greater than zero, so we will consider the stacking to be a mixture of each type of stacking. It might seem easier to consider the entire volume of the bus and compare that to the volume of a ping pong ball to calculate the number of balls to fill the bus. However, because of stacking nuances and the discrete nature of ping pong balls this method breaks down, and each dimension needs to be considered separately

Sections:

I. Formulating the number of balls in a Simple Cubic stacking.

In each dimension, the next nearest ball is a distance of 2r away. It might seem easier to consider the entire volume of the bus and compare that to the volume of a ping pong ball to calculate the number of balls to fill the bus. However, because of stacking nuances and the discrete nature of ping pong balls this method breaks down, and each dimention needs to be considered separately.

1. Calculate the number of balls in each direction (Where x,y,&z are the dimensions of the bus and each b(x,y, and z) represent the number of balls that fit along that axis).

$$b_x = \frac{X}{2r} \qquad b_y = \frac{Y}{2r} \qquad b_z = \frac{Z}{2r}$$

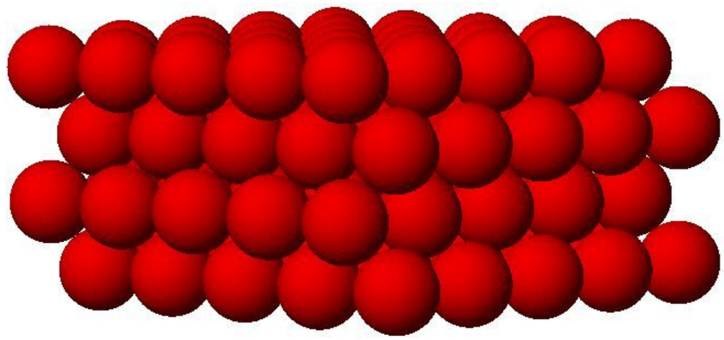
2. Round down to get a count for the number of whole balls that actually fit in each dimension.

Example: 204.3 → 204, 105.9 → 105

3. Calculate the total number of balls that fit in a strictly Simple Cubic stacking.

$$B_T = b_x * b_y * b_z$$

II. Formulating the number of balls in a Hexagonally Close Packed stacking.



1. Calculate the number of balls in each direction.

The calculation for the number of balls in the X dimension is the same as in Simple Cubic stacking…

$$b_x = \frac{X}{2r}$$

However, this changes in Y and Z because the balls in each new layer are nested in between the balls of the previous layer. More balls fit within the same Y and Z dimensions because of this nesting (or alternatively, each ball takes up less Y and Z space). In Y: each new row of balls adds $r\sqrt{3}$ to the Y dimension (note this is less than the 2r that gets added to each dimension in Simple Cubic stacking.

Example: Distance after each new row in Y

First Layer	2r
Second Layer	2r + $r\sqrt{3}$
Third Layer	2r + $2r\sqrt{3}$

# of balls in Y : $b_{yA} = \frac{Y - 2r}{r\sqrt{3}} + 1$

In Z: each new layer adds $\frac{2r\sqrt{6}}{3}$.

Example: Distance after each new layer in Z

First Layer	2r
Second Layer	2r + $\frac{2r\sqrt{6}}{3}$
Third Layer	2r + $\frac{4r\sqrt{6}}{3}$

$$\text{\# balls in Z : } b_z = \frac{3(Z - 2r)}{2r\sqrt{6}} + 1$$

In hexagonal packing, the layers follow an alternating ABABAB… pattern.
So far the calculations have been mostly regarding the A layer.
For this reason we also need to find the number of balls which fit in the Y direction in the B layer.

The starting point for the first ball in the B layer is no longer 2r. It has shifted to

$$r + \frac{r\sqrt{3}}{3}$$

Like the A layer, they still increase by an increment of $r\sqrt{3}$.

$$\text{\# of balls in Y of B layer : } b_{yB} = \frac{3Y - 6r - r\sqrt{3}}{3r\sqrt{3}} + 1$$

2. Calculate the number of balls in each layer.

If the remainder of $b_x \geq .5$:

$B_A = b_x * b_{yA}$

$B_B = b_x * b_{yB}$

If the remainder of $b_x < .5$:

Calculate sheet A:

If b_{yA} is even:

$$B_A = b_x * b_{yA} - \frac{b_{yA}}{2}$$

if b_{yA} is odd:

$$B_A = b_x * b_{yA} - \frac{b_{yA} - 1}{2}$$

Calculate sheet B:

If b_{yB} is even:

$$B_B = b_x * b_{yB} - \frac{b_{yB}}{2}$$

If b_{yB} is odd:

$$B_B = b_x * b_{yB} - \frac{(b_{yB} - 1)}{2}$$

3. Calculate the total amount of balls:

If b_z is even: $B_{Total} = (B_A + B_B) * \frac{b_z}{2}$

If b_z is odd: $B_{Total} = (B_A + B_B) * \frac{b_z}{2} + B_A$

III. Calculations for the school bus

The bus model we will use is the *Thomas Saf-T-Linder HDX*.

The given dimensions:

Height (**Z**) : 78” (1,981.2 mm) *(www.thomasbus.com/_pdf/brochure-hdx-school-2010.pdf*
Width (**Y**) : 96” (2,438.4 mm) *(www.wikipedia.org/wiki/Thomas_Saf-T-Liner)*
Length (**X**) : 40 ft. (12,192 mm) *(Legally a school bus can not be longer than 45 ft. in total length. This bus is quite large and seats 90 children, so I assume it is near the max length. Reducing it a bit to account for the bumpers, I settled with 40 ft.)*

Radius of standard ping pong ball (**r**): 20mm *(www.tabletennismasters.com/ping-pong-balls.html)*

The radius of the ping pong ball is the most exact measurement among the rest of the dimentions, so we will be using millimeters as our standard unit in the calculations.

1. Solving for Simple Cubic

Plugging in these values to solve for b_x , b_y , b_z :

$$b_x = \frac{X}{2r} = 304.8 \qquad b_y = \frac{Y}{2r} = 60.96 \qquad b_z = \frac{Z}{2r} = 49.53$$

Round down the values to get the number of whole balls that fit.

$$b_x = 304 \qquad b_y = 60 \qquad b_z = 49$$

Get the total number of balls:

$$B_T = b_x * b_y * b_z = 893{,}760 \; ping \; pong \; balls$$

2. Solving for Hexagonally Close Packed:

Plugging these values to solve:

$$b_x = \frac{X}{2r} = 304.8 \qquad b_{yA} = \frac{Y-2r}{r\sqrt{3}} + 1 = 70.2$$

$$b_z = \frac{3(Z-2r)}{2r\sqrt{6}} + 1 = 60.4 \qquad b_{yB} = \frac{3Y-6r-r\sqrt{3}}{3r\sqrt{3}} + 1 = 69.9$$

(after rounding down $b_x \rightarrow 304$, $b_{yA} \rightarrow 70$, $b_z \rightarrow 60$, $b_{yB} = 69$)

Since the remainder of $b_x \geq 0.5$:

$$B_A = b_x * b_{yA} = 21{,}280 \qquad B_B = b_x * b_{yB} = 20{,}976$$

Since b_z is even:

$$B_{Total} = (B_A + B_{B)} * \frac{b_z}{2} = 1{,}267{,}680 \text{ ping pong balls}$$

Therefore it will take about 893,760 – 1,267,680 ping pong balls to fill the bus.

IV. Further Thoughts

Often, with riddle-like, open ended or generalized questions there is room for multiple answers. Here are some alternatives (mostly trivial).

A riddler might answer:

> You can never "fill" the bus because there will always be gaps between the ping pong balls.
>
> Or, if the windows are open, you will never be able to fill the bus.
>
> Or, none if the bus is already filled with marbles.

An experimentalist may try seeing how many ping pong balls fit into a smaller cuboid (like a shoebox) and then extrapolate to the size of the bus.

A theoretical physicist would find the solution to a spherical bus in a vacuum.

~ Thank you for reading :)